Focusing Vacuum Fluctuations II

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Abstract

The quantization of the scalar and electromagnetic fields in the presence of a parabolic mirror is further developed in the context of a geometric optics approximation. We calculate the mean squared scalar and electric fields near the focal line of a parabolic cylindrical mirror. These quantities are found to grow as inverse powers of the distance from the focus. We give a combination of analytic and numerical results for the mean squared fields. In particular, we find that the mean squared electric field can be either negative or positive, depending upon the choice of parameters. The case of a negative mean squared electric field corresponds to a repulsive Van der Waals force on an atom near the focus, and to a region of negative energy density. Similarly, a positive value corresponds to an attractive force and a possibility of atom trapping in the vicinity of the focus.

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1 Introduction

In a previous paper [1], henceforth I, we developed a geometric optics approach to the quantization of scalar and electromagnetic fields near the focus of a parabolic

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mirror. We found that there can be enhanced fluctuations near the focus in the sense that mean squared field quantities scale as an inverse power of the distance from the focus, rather than an inverse power of the distance from the mirror. These enhanced fluctuations were found to arise from an interference term between different reflected rays. In the present paper we extend the previous treatment. In I, only points on the symmetry axis were considered. Here we are able to treat points in an arbitrary direction from the focal line of a parabolic cylinder. We give more detailed numerical results which provide a fuller picture of the phenomenon of focusing of vacuum fluctuations. We also correct some erroneous results in Sect. V of I.

The outline of the present paper is as follows: In Sect. 2, we review and extend some of the formalism used to calculate the mean squared scalar and electric fields, $\langle\varphi^2\rangle$ and $\langle\mathbf{E}^2\rangle$, near the focus. In Sect. 3 we derive some geometric expressions which are needed to study fluctuations at points off of the symmetry axis. In Sect. 4, the evaluation of integrals with singular integrands is revisited. Two different, but equivalent, approaches are discussed. A particular case where the integrals can be performed analytically is examined in Sect. 5. More generally, it is necessary to calculate $\langle\varphi^2\rangle$ and $\langle\mathbf{E}^2\rangle$ numerically. A procedure for doing so is outlined in Sect. 6. Some detailed numerical results are also presented there. The limits of validity of our model and results will be examined in Sect. 7. This discussion will draw on some results on diffraction obtained in the Appendix. The experimental testability of our conclusions will be discussed in Sect. 8. Finally, the results of the paper will be summarized in Sect. 9.

Units in which $\hbar = c = 1$ will be used throughout this paper. Electromagnetic quantities will be in Lorentz-Heaviside units.

2 Basic Formalism

Here we will briefly review the geometric optics approach developed in I. The basic assumption is that we may use a ray tracing method to determine the functional form of the high frequency modes, which will in turn give the dominant contribution to the expectation values of squared field operators. We start with a basis of plane wave modes. The incident wave, for a scalar field, may be taken to be

$$f_{\mathbf{k}} = \frac{1}{\sqrt{2\omega V}}\, \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}\,, \tag{1}$$

with box normalization in a volume V. In the presence of a boundary, this is replaced by the sum of incident and reflected waves,

$$F_{\mathbf{k}} = f_{\mathbf{k}} + \sum_i f_{\mathbf{k}}^{(i)}\,, \tag{2}$$

where the $f_{\mathbf{k}}^{(i)}$ are the various reflected waves. One could also in principle adopt a wavepacket basis, in which $F_{\mathbf{k}}$ is replaced by a localized wavepacket. Because the time evolution preserves the Klein-Gordon norm, if the various modes are orthonormal in the past, they will remain so after reflection from the mirror. Thus we can view Eq. (2) as the limit of a set of orthonormal wavepacket modes in which the modes become sharply peaked in frequency and hence delocalized.

It was shown in I that the renormalized expectation value of the squared scalar field is given by a sum of interference terms:

$$\langle \varphi^2 \rangle = \sum_{\mathbf{k}} \left[\sum_i (f^*_{\mathbf{k}} f^{(i)}_{\mathbf{k}} + f_{\mathbf{k}} {f^{(i)}}^*_{\mathbf{k}}) + \sum_{i \neq j} f^{(i)}_{\mathbf{k}} {f^{(j)}}^*_{\mathbf{k}} \right] . \tag{3}$$

This renormalized expectation value is defined as a difference in the mean value of φ^2 with and without the mirror, and hence will vanish at large distances from the mirror. The various interference terms in the above expression yield contributions to $\langle \varphi^2 \rangle$ which are proportional to the inverse square of the appropriate path difference. Thus in the vicinity of the focus, the interference terms between different reflected rays will dominate over that between the incident and a reflected ray. In the present paper, we will consider cases with no more than two reflected rays, and write

$$\langle \varphi^2 \rangle \approx 2\, \mathrm{Re} \sum_{\mathbf{k}} f^{(1)}_{\mathbf{k}} {f^{(2)}}^*_{\mathbf{k}} . \tag{4}$$

In the case of a parabolic cylinder, this may be expressed as

$$\langle \varphi^2 \rangle = -\frac{1}{3\pi^2} \int \frac{d\theta'}{(\Delta\ell)^2} . \tag{5}$$

Here $\Delta\ell$ is the path difference for the two reflected rays, and the integration is over the reflection angle of one of the rays. The range is chosen so that each pair of reflected rays is counted once. The corresponding expression for the mean squared electric field near the focus of a parabolic cylinder is found in I to be

$$\langle \mathbf{E}^2 \rangle = \frac{8}{5\pi^3} \int \frac{d\theta'}{(\Delta\ell)^4} . \tag{6}$$

3 Optics of Parabolic Mirrors

In this section, we wish to generalize some of the results of I concerning the incident and reflected rays in the presence of a parabolic mirror. Consider the geometry illustrated in Fig. 1. An incident ray at an angle of θ to the symmetry axis is reflected from the point (x_i, y_i), and then reaches the point P at an angle of θ'. We first need to find the relation between θ and θ'. Note that the reflected ray crosses the symmetry axis at a distance c from the focus. It was shown in I that

$$\theta = \frac{c \sin^3 \theta'}{b(1 - \cos\theta')} . \tag{7}$$